Calculations of Error Structures

in Binary Channels with Memory

BSC-M as an extension of the BSC Channel

First edition 2018

Claus Wilhelm

Universities teaching Communication Technology are certain to benefit from applications of the BSC-M Model, for the first time facilitating the evaluation of transmission procedures and codes.
This book is also written for those interested in in development and testing of link control protocols and error protection schemes for communication systems. The content is thereby intended to offer a full set of calculations of error structures occurring within binary channels with memory (BSC-M).

For this purpose it is not necessary to measure error structures.The model contains only two parameters, the bundling factor and the bit error rate, whereby the most diverse range of channels can be presented, derived from empirical readings conducted over twenty-three years. It have been verified by different Authors, as well as by measurements over cable and also over maritime radio up to distances of 7,500 km (short-wave).

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PREFACE

This book is written for those interested in in development and testing of link control protocols and error protection schemes for communication systems. The content is thereby intended to offer a full set of calculations of error structures occurring within binary channels with memory (BSC-M).

The new approach to calculating error structures using the *A-Model* based on its predecessor the *L-model* is derived from empirical readings conducted over twenty-three years, from 1966-1989, based on a vast number of channels with burst errors, thousands of measuring hours in channels with 200 bit/s up to 2,048 Mbit/s with varying error structure measuring devices.

These models have been verified by different Authors, as well as by measurements over cable and also over maritime radio up to distances of 7,500 km (short-wave).

The current teaching material at universities is dominated by work concerning binary channels without memory whereby the easiest models typically involve:

Binary Symmetric Channels (BSC),
Additive White Gaussian Noise (AWGN).

These models do not contain burst structures, as an unrealistic assumption. On the other hand, intuitively derived mathematical models exist with which one can attempt to adapt measured readings, for instance E. N. Gilbert's Model[1] from 1960, commonly termed *Hidden Markoff Models (HMM)* as per WilliamTurin.[2] Therefore by using Markoff Models, burst structures can be simulated.

However the distance distribution between bursts cannot be simultaneously simulated as Markoff Models comprise a limited number of states based on finite samples (an example of the fuzzy problem).

Calculations of error structures using these models are rarely found in the relevant literature. One example is the calculation of the probability that g bit errors will be found in a block of length n.

The *A-Model* is based on a distance function between bit errors, which is similarly found in a large variety of channels. The model contains only ***two parameters*****,** the bundling factor and the bit error rate, whereby the most diverse range of channels can be presented.

Despite the simplifying assumption that the *A-Model* is a renewal model*,* by which the memory only reaches the last error symbol, the resulting calculations of probabilities prove to approximate real measured values. With the help of such models, closed analytical expressions for error structures of bursts and blocks can be developed. An example is the probability of g errors occurring in bursts or in blocks of length n, or the probability of burst lengths and error patterns.

As a result, the model enables both the comparison of different codes and procedures with one another as well as the optimization of communication systems. It is sufficient to vary the two parameters. For this purpose ***it is not necessary*** to measure error structures.

Universities teaching Communication Technology are certain to benefit from applications of the *A-model,* for the first time facilitating the evaluation of transmission procedures and codes.
On introduction is presented in teaching materials of LNTwww, TU München, from 2015 [14].

For college students, professors and commercial developers finally able to make such evaluations more simply, an entire new realm of possibilities for future problem solving – such as in Machine-to-Machine (M2M) Communication – will come into being. Until now we have been limited to empirical tests, based on already standardized protocols concerning different channels whereby we required no more than a minimum quality for such channels.

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[1] Gilbert, E N: *"Capacity of a Burst-Noise Channel"* in Bell Systems Technical Journal 39 (1960/II), pp 1253-1265.
[2] Turin, William: *"Performance Analysis and Modeling of Digital Transmission Systems"* 2004, Kluwer Academic / Plenum Publishers.

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Channel models and methods for analysis and synthesis of modern digital transmission media, Distance model (A-model) and gap model (L-model)

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1. Two new models of the binary symmetric channel with memory, the A-model and the L-model for teaching and research

The symmetric binary channel with memory can be represented by so-called burst models [1][2] which satisfy specified accuracy requirements. In this case, the memory length is a function of the memory content. The construction of such models is possible by statistical analysis of measured error sequences [3] [4] with special error structure measuring equipments (Fig. 1).

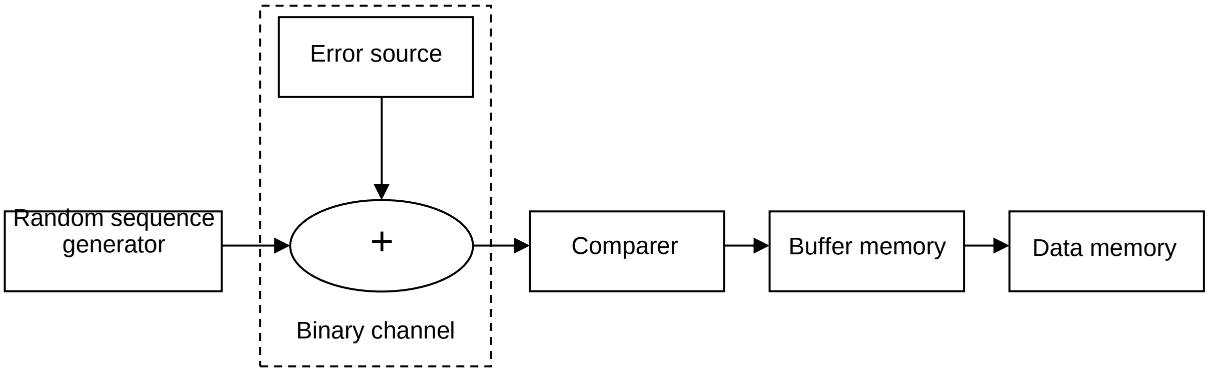


Figure 1 *: Principle of the error structure measurement*

The measurement of error-sequences with the required measuring devices is expensive and time consuming. But for theoretical studies in teaching and research such models are needed for the symmetric binary channel with memory models must be sufficiently precise and mathematically as simple as possible for users. For this reason, we introduce two models, the L-model (gap model) and the A-model (distance model). They form the basis of the clear presentation and calculation of a large number of practically occurring binary channels. In particular, the application of the A-model allows the calculation of interesting error structures in bursts and blocks using relatively simple formulas. In the following proposed models, the memoryless channel is included as a borderline case. Therefore, it should be dispensed with in most cases, to enable the meaningful use of a model of the binary symmetric channel without memory as the basis of classical considerations.

1.1 Channel disturbance and interruptions

Each received signal which has been transmitted through a disturbed channel of some description, will, at best, be only similar to the original signal, but not identical. The amount of the expected signal forms should therefore be chosen so that the probabilities for the random transformation of valid signals into each other be as small as possible. By sampling the amplitude values (symbol recognition) and by comparing them with a reference value, the transmitted binary signals will be as safe as possible in the receiver. The sampling needs to be located within the symbol. The time interval for the symbol detection through synchronization of the demodulated signals is thereby derived. It is presumed that the channel is interrupted if the synchronization between transmitter and receiver is lost and that during the interruption the synchronisation cannot be restored. In the following models, interruptions are neglected and intervals are seamlessly joined together (Fig.2).

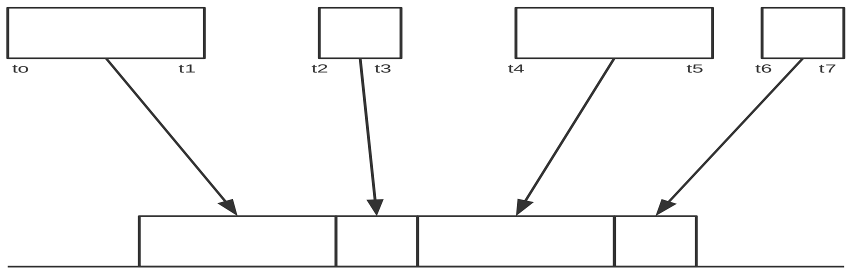
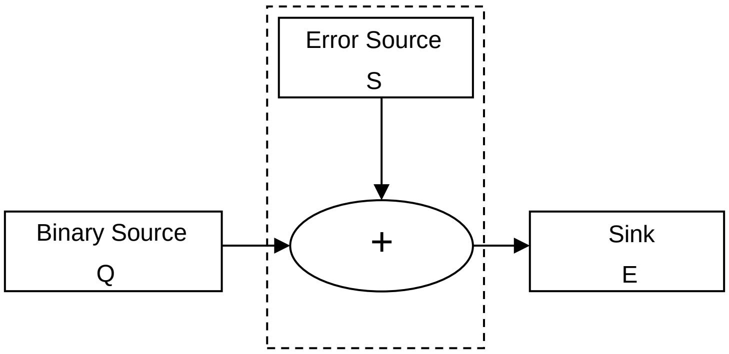


Figure 2:*Sequence of the disturbed and interrupted transmission intervals*

In the symmetric binary channel, the symbol error generated by the error source S is independent of the symbols of the source Q (Fig. 3).



Q	S	E	
L	O	L	
L	L	O	error
O	O	O	
O	L	L	error

Figure 3*:* *Model of binary symmetric channel*

The so-defined channel is fully described by the timed order of the occurrence of symbol errors.

1.2 Empirically determined channel properties

Each modelled error sequence contains k error-free symbols between any two symbol errors.

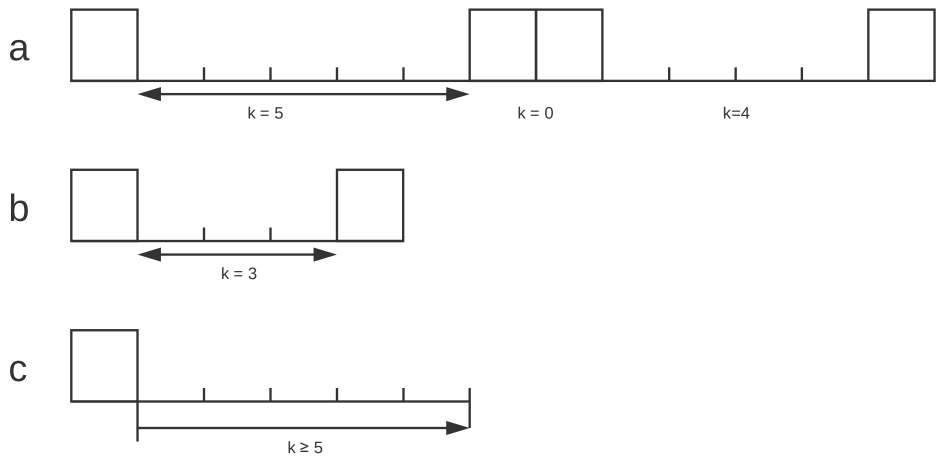


***Figure 4**: Parameters for the description of the modeled error sequence*

Given the probability $v(k)$, the distance to the next symbol error is exactly k error-free steps (Fig.4b). The probability $u(k)$ that after a symbol error of at least k errorless steps follows (Fig. 4c), is the error distance distribution with

$$u(0) = 1$$

$$\lim_{k \to \infty} u(k) = 0$$

$$\mathrm{v}(k) = u(k) - u(k+1)\,. \tag{1}$$

If the data are transferred in blocks of length n, these blocks are incorrect with probability $p_b(n)$

The block error probability is a function of block length n**.** Figure 5 shows these functions in double-logarithmic presentation.

In a variety of analyses, the fact that the presented functions show linear increases from about 0.5 to 0.95 in the initial part, was empirically confirmed several times over. This applies for all previously evaluated measurements in the range of 50 bit/s to 2.048 Mb/s. For the asymptotes shown in Figure 5, the linear equation applies

$$\log[p_n(n)] = \log p_e + \alpha \cdot \log n$$

It follows

$$p_b(n) = p_e \cdot n^{\alpha} \tag{2}$$

where p_e is the symbol error probability, and $(1-\alpha)$ is the bundling factor where

$0 \le (1-\alpha) \le 0.5$.

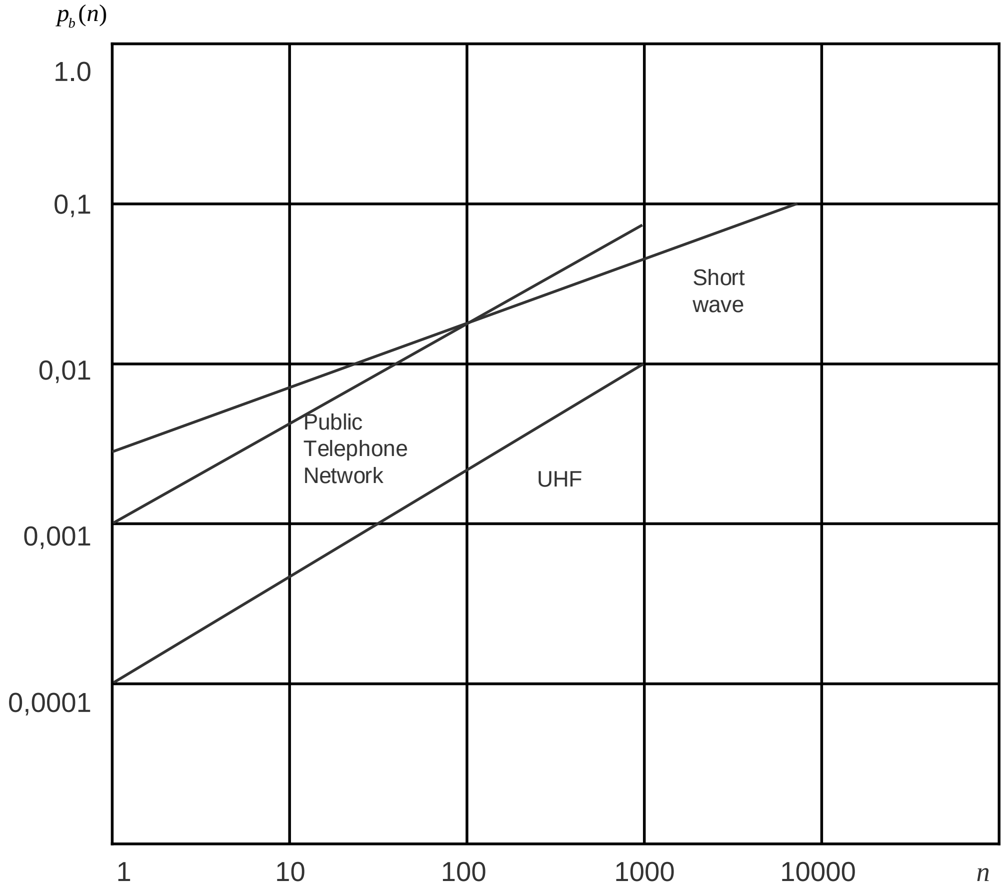


***Figure 5**: Block error probability as a function of the block length (examples)*

1.3 Derivation of the error distance distribution of the L-model

From Figure 6 one can see how the block error probability may be calculated from the error distance distribution.

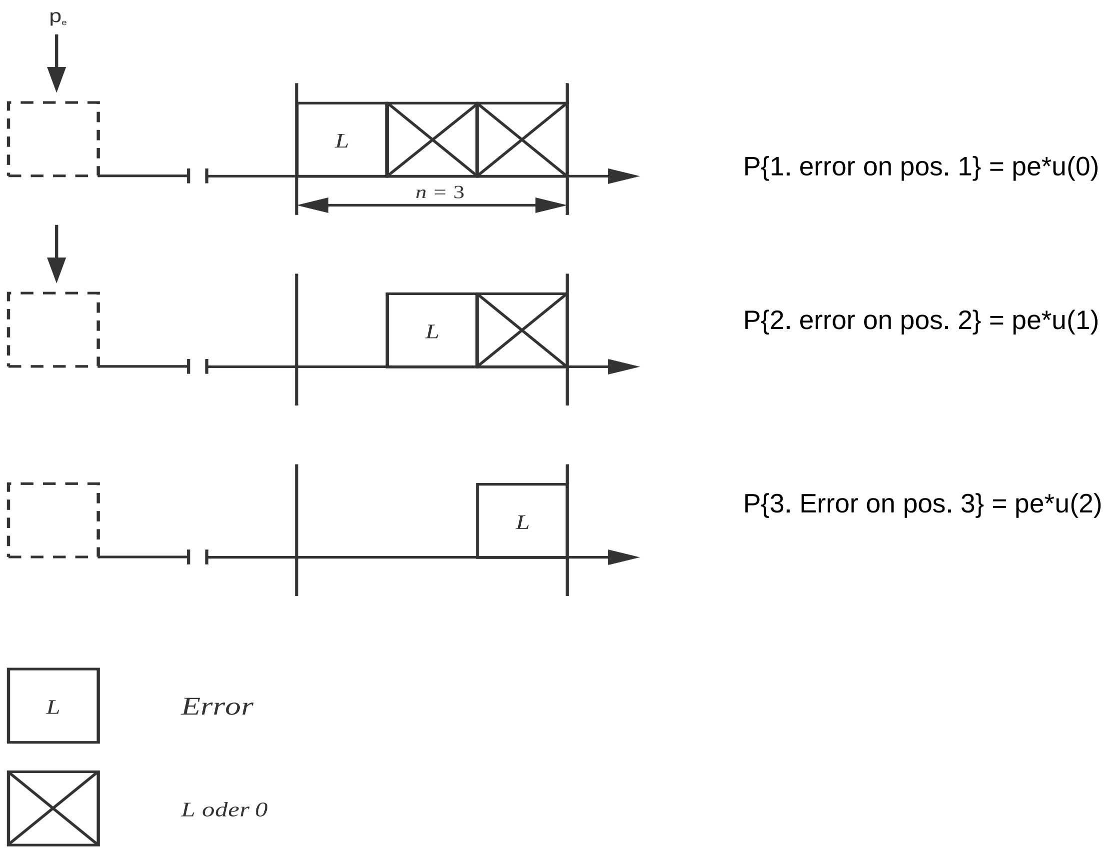


Figure 6: *Calculation of* $p_b(n)$ *from* $u(k)$

The probability that the first error bit in block of length n is located at some position, is $p_e \cdot u(k)$, where k is the number of error free steps in front of this symbol error. By totaling all n positions we obtain the following result:

$$p_b(n) = p_e \cdot \sum_{k=0}^{n-1} u(k) \tag{3}$$

From the empirically found asymptote for p_b we then obtain an asymptote for $u(k)$:

$$p_b(n) = p_e \sum_{k=0}^{n-1} u(k) = p_e \cdot n^{\alpha}$$

For n = 1, 2, 3,….. we obtain by substitution

$$u(0) = 1^{\alpha}$$

$$u(0) + u(1) = 2^{\alpha}$$

$$u(0) + u(1) + u(2) = 3^{\alpha}$$

It follows then, for the desired asymptote

$$u(k) = (k+1)^{\alpha} - k^{\alpha} \tag{4}$$

However, as the asymptote in accordance with (2) for large block length n values of $p_b(n) > 1$ is achieved, then $u(k)$ s unsuitable as a distribution function according to (4). Therefore it must be

$$\lim_{n\to\infty} p_e \cdot \sum_{k=0}^{n-1} u(k) = 1 \quad \text{and} \tag{5}$$

$$\lim_{n\to\infty} p_b(n) = 1$$

To meet the additional condition of (5), $u(k)$ is empirically multiplied with the asymptote $e^{-\beta k}$

$$u(k) = [(k+1)^{\alpha} - k^{\alpha})] \cdot e^{-\beta k} \tag{6}$$

Here $\beta > 0$ The parameter β must be determined so that equation (5) is satisfied.

Using the approximate generating function, from which the later A-model has also been developed, one obtains according to formula (24):

$$e^{-\beta} = C = (1 - p_e^{1/\alpha}) \tag{7}$$

Note that the block error probability $p_b(n)$ remains constant when the gaps between the symbol errors are rearranged, so (3) remains unaffected. It follows that for a given distance distribution $u(k)$, the in function $p_b(n)$ remains unchanged whether successive gaps occur dependently or independently of one another. Therefore, it is assumed for the sake of simplicity, that the gaps are independent.

Definition 1: The L-model (gap model) of the binary symmetric channel with memory contains successively independent gaps of length k between the individual error symbols. The probability $u(k)$ that after a symbol error k or more error-free symbols follow, is calculated from

$$u(k) = \left[(k+1)^{\alpha} - k^{\alpha}\right] \cdot C^{k} \qquad \text{where } C = e^{-\beta} = (1 - p_e^{1/\alpha}), \text{ from formula (24)} \tag{8}$$

where $u(0)=1$; $\lim_{k\to\infty} u(k) = 0$; k is the number of error free steps between two symbol-errors, $(1-\alpha)$ is the bundling factor in the Channel $0,5 \le \alpha \le 1$

In this model, the channel-memory extends back to the previous symbol error in each case.

The gap lengths k are independently and identically distributed random variables. The symbol errors are **recurrent events**, whereby Markoff's chains occurring within bursts are dispensed with.

1.4 Derivation of the generating function and the distribution of error distance in the A-model

The L-model is used to calculate the probability of the occurrence of error patterns within a time interval Δt Each error pattern is one of 2^b elementary events which can be observed within a time segment of bits length b .

However, for practical use of channel models, it is necessary to calculate the probabilities for the occurrence of error patterns. Error structures are subsets of error patterns which have the same predetermined characteristics, such as the number g of symbol errors, the length l of the burst error, etc. To calculate the probabilities of such sums of random variables, the so-called generating function should be employed. In this specific case, the gap length k is our random variable; $u(k)$ is designated as u_k , the probability between any two symbol errors, when the gap is equal to or more then k bits. Then

$$U(t) = u_0 + u_1 t + u_2 \cdot t^2 + \ldots = \sum_{k=0}^{\infty} u_k \cdot t^k \tag{9}$$

is the generating function of the error distance distribution $u(k)$. If the integers k_1 and k_2, occur as mutually exclusive and independent, as well as non-negative random variables with generating functions $U_1(t)$ and $U_2(t)$, then their sum k_1+k_2 is the generating function $U_1(t)\cdot U_2(t)$:

$$C(t)=\sum_{k=0}^{\infty} C_k \cdot t^k = U_1(t)\cdot U_2(t) \tag{10}$$

Modelling *Data transmission via*	*Error probability* p_e	*Bundling factor* $(1-\alpha)$
Short wave	$5\cdot 10^{-2} > p_e > 5\cdot 10^{-3}$	$0.5>(1-\alpha)>0.25$
Automatic Phone Channels (medium speed, mech. Selectors)	$10^{-2} > p_e > 10^{-4}$	$0.3>(1-\alpha)>0.1$
Automatic Phone Channels (200 Bd; mech. selectors)	$10^{-3} > p_e > 10^{-5}$	$0.3>(1-\alpha)>0.1$
Leased Line, 48 kbit/s with Modem	$10^{-5} > p_e > 10^{-6}$	$0.3>(1-\alpha)>0.1$
PCM-Channels with *2,048 Mbit/s*	$10^{-7} > p_e > 10^{-9}$	$0.2>(1-\alpha)>0$

***Table 1**: Typical values for the parameters of A-model and L-model for estimated calculations*

The Taylor series expansion as in (17), leads to the new approximated error distance distribution based on the L-model (6)

$$u(k)=\frac{\alpha\cdot(1+\alpha)\cdot\ldots\cdot(k-1+\alpha)}{k!}\,\mathrm{e}^{-ßk} \tag{19}$$

for k = 1,2 ... with u (0) = 1. As given in [5], the following equations apply to the Γ-Funktion.

$$\alpha(1+\alpha)(2+\alpha)\cdot\ldots\cdot(k-1+\alpha)=\frac{\Gamma(\alpha+k)}{\Gamma(\alpha)}$$

$$k!=\Gamma(k+1)$$

for $k=1,2,\ldots,$ with $0!=\Gamma(1)=1, 1!=\Gamma(2)=1$.

$$u(k)=\frac{\Gamma(\alpha+k)}{\Gamma(1+k)\cdot\Gamma(\alpha)}\cdot \mathrm{e}^{-ßk} \quad \text{with} \tag{19a}$$

$e^{-\beta}=(1-p_e^{1/\alpha})=C$, derived from formula (24)

Definition 2: A channel model with independently successive gaps between symbol errors which has the distribution function for the error distances

$$u(k)=\frac{\alpha(1+\alpha)\cdot\ldots\cdot(k-1+\alpha)}{k!}\cdot \mathrm{C}^k \quad \text{whereby } k=1,2,3,\ldots \tag{20}$$

whereby $C=(1-p_e^{1/\alpha})$ according to formula (24)

Here, k is the distance between two symbol errors, and $u(0)=1$, and p_e the symbol error probability and the clustering factor; According to the formula (24), the A-model includes the memory-free BSC channel on account of $u(k;\alpha=1)=(1-p_e)^k$

Recursively $u(k)$ can be calculated thus: $u(k+1)=u(k)\cdot\frac{k+\alpha}{k+1}\cdot C$ with $u(0)=1$.

And the generating function of $u(k)$ is $U(t)=\sum_{k=0}^{\infty}u(k)\cdot t^k=\frac{1}{(1-C\cdot t)^{\alpha}}$

1.5 Symbol error probability and the block error probability

The reciprocal of the symbol error probability p_e is calculated from the average error distance $E(k)$:

$$\frac{1}{p_e}=E(k)+1$$

$$E(k)=\sum_{k=0}^{\infty}k*v(k)*t^k=(\sum_{k=0}^{\infty}u(k)*1^k)-1=U(1)-1 \tag{23}$$

$$p_e=\frac{1}{U(1)}=(1-e^{-\beta})^{\alpha} \quad \text{whereby } p_e^{1/\alpha}=1-e^{-\beta} \text{ and } C=e^{-\beta}=(1-p_e^{1/\alpha}) \tag{24}$$

The block error probability is, see (8), (20):

$$p_b(n)=p_e\cdot\sum_{k=0}^{n-1}u(k)$$

For the BSC Channel (without of memory), where $\alpha = 1$ results for the L-model and the A-Model:

$$p_b\left(n, \alpha = 1\right) = p_e \cdot \left(u(0) + \sum_{k=1}^{n-1} u(k) \right) = p_e \cdot \left(1 + \sum_{k=1}^{n-1} C^k \right) = p_e \cdot \frac{1-(1-(1-p_e)^n}{1-(1-p_e)} = (1-p_e)^n$$

1.6 Probability $v(k)$ of the gap length k in the A-model

Because $v(k) = u(k) - u(k+1)$, it follows from (19)

$$v(k) = u(k)\left(1 - \frac{k+\alpha}{k+1} \mathrm{e}^{-ß}\right) \tag{25}$$

$$v(k) = u(k)\left(1 - \frac{k+\alpha}{k+1} \cdot \mathrm{C}\right) \tag{26}$$

Fort the memoryless BSC-channel with $\alpha = 1$ it results

$$v(k; \alpha = 1) = u(k) * p_e = p_e(1-p_e)^k$$

1.7 Symbol correlation function $r(s)$ for the A-Model

In various cases of practical interest, the probability of error structures should be to calculated whereby one symbol error occurs exactly s steps after another symbol error. Whether or not the intervening symbols are also wrong is not relevant (Figure 7). An example is the calculation of the probability of the length of error bundles in erroneous blocks. For these applications, the correlation probability $r(s)$ is determined for the A-model when two symbol errors occur at a distance s at the points i and $i+s$ of the error sequence: $r(s) = P\{x_i = L; x_{i+s} = L\}$

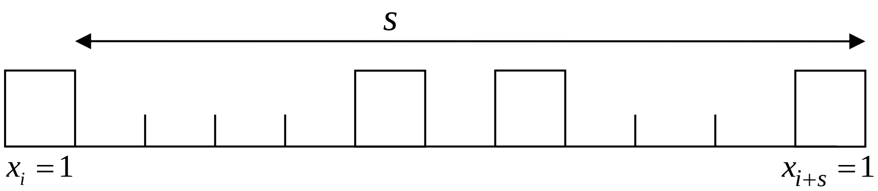


Figure 7: *Calculation of the correlation probability r (s) for the occurrence of two symbol errors at a distance of* S

For our calculation we denote the probability $p(s) = v(s-1)$ for the first return of an error after exactly s steps at a distance of s with $p(s) = p_S$. For the n^{th} return of a symbol error we write $p_s^{(n)}$

where $p_s^{(n)} = 0$ for $n > s$. This gives us:

$$r(s) = p_e \cdot (p_s^{(1)} + p_s^{(2)} + ...) \tag{27}$$

Multiplied by t^s and summed over all s we get the generating function for the correlation probability:

$$R(t) = \sum_{s=1}^{\infty} r(s)t^s = p_e(\sum_{s=1}^{\infty} p_s^{(1)}t^s + \sum_{s=1}^{\infty} p_s^{(2)}t^s + ...) \tag{28}$$

Now the correlation probability for 0 steps is added

$$r(0) = p_e \cdot 1$$
$$p_0^{(1)} = p_0^{(2)} = ... = 0$$

And we apply the convolution theorem:

From this the generating function R (t) for the A-model:

$$R(t) = p_e \cdot \frac{(1 - C \cdot t)}{1 - t} \tag{32}$$

$$\begin{aligned} r(0) &= p_e \cdot 1, \\ r(1) &= p_e(1 - \frac{\alpha \cdot C}{1!}), \\ r(2) &= p_e(1 - \frac{\alpha}{1!}C - \frac{\alpha(1-\alpha)}{2!}C^2 \\ r(s) &= p_e(1 - \frac{\alpha}{1!}C - \frac{\alpha(1-\alpha)}{2!}C^2 - .... - \frac{\alpha(1-\alpha)....(s-1-\alpha)}{s!}C^S \end{aligned} \tag{35}$$

as the correlation probability for the A-model.

For the symmetrical binary channel without memory, the result with $\alpha = 1$ is as expected:

$$r(s) = p_e(1 - C) = p_e(1 - (1 - p_e^{1/\alpha})) = p_e^2$$

k	*u(k)*	
	measured [6]	*Calculated according to equations (30), (22), (23)*
1	*0.8409*	*0.8099*
5	*0.6393*	*0.6287*
100	*0.3582*	*0.3582*
500	*0.2492*	*0.2457*
1000	*0.2073*	*0.1986*

Table 2: *Error distance distribution* $u(k)$*, measured and calculated with the A-model as the probability that the gap length between two symbol errors is* $\geqq k$

1.10 Summary

Using the two presented channel models with independent gaps between successive symbol errors, theoretical assessments of the effectiveness of data transfer procedures and encodings are possible.

According to the given distribution function one can create artificial error sequences and sort, or calculate analytically, their probability for error structures.

With both these models, the course of the block probability is reflected exactly asymptotically along the block length. Both models are significantly more accurate than the previously proposed models with independent gaps between consecutive errors. They are even more exact than the well-known principal models of other authors that have been in use for a long time, including Gilbert (1960, [7]).

In conclusion, error structures are presented in Table 3 for comparison between measured and calculated frequencies of A-model error structures.

A **burst** begins with a symbol error, and is completed when a error-free consecutive steps follow. A **bundle** in a block begins with the first symbol error and ends with the last symbol error.	Parameter	Example: Error structures in a public Strowger switch telephone network with failure-prone mechanical switches, modem 200 Bd, 16 Hours, 9807 symbol errors $p_e = 0.0008488$		
		Measured number	Number calculated with A-model (α = 0.84)	Formula No. from download (see Channel model part 2 of the homepage)
Error structures in bursts with the distance parameter a 1)				
Number z_B of bursts in the sample	a=1 a=2 a=5	8247 6270	8238 7577 6659	(37)
Mean number of errors E {g} in bursts	a=5	1.56	1.47	(38)
Number of bursts with g errors	a=5; g=4	not analyzed	149	(40)
Number of bursts of length l	a=2; l=5	not analyzed	61	(57)
Error structures in blocks of length n				
Number of blocks with single errors (g = 1)	n=5 n=1023	1096 6271	940 6343	(67)
Number of blocks with error bundle of length $l > 3$	N=5; l >3	426	319	(72)
Number of blocks with error bundle of length l = n and g errors	l =n=5; g=3	45	35	(90)
Number of blocks of length n with error bundle of length l = n and g errors	l =5; g=3; n=63	18	48	(95)

1) A burst begins with a symbol error and is completed when error-free consecutive steps follow

Table 3: Comparison of measured and frequencies of error structures calculated with the A-model

2. Calculation of error structures in bursts

2.1 Number of bursts

A burst begins with a symbol error and ends when at least a error free steps follow each other (see Fig. 8). Here a is referred to as the distance parameter. The error structures within bursts and the number of bursts are especially of interest with respect to block-free codes [2], [8].

Hereafter the number z_B of bursts is calculated within a sample with z_f symbol errors and the distance parameter a. When the gap for a symbol error is equal to or greater than a, the burst is terminated.

The proportion of these gaps is $u(a)$. The results, regardless of the memory properties of the channel, for each sample yield:

$$z_B = z_f * u(a)$$

The number of bursts remains constant, for all of the permutations of the successive error intervals and for all the error-distance distributions $u(k)$, for which $u(a)$ has the same value.

Especially with respect to (20) in Part 1, the following applies for our A-model

$$z_B = z_f \bullet \frac{\alpha \cdot (1+\alpha) \cdot \ldots \cdot (a-1+\alpha)}{a!} \cdot \mathrm{C}^a \quad \text{where} \quad C = (1 - p_e^{1/\alpha}) \tag{37}$$

It is evident that the average number $E(g)$ of errors in a burst is therefore easy to calculate:

$$E\{g\} = \frac{z_f}{z_B} = \frac{1}{u(a)} \tag{38}$$

Example 1:
Calculate the number z_B of the bursts for the A-model, for a given sample where $z_f = 9807$ and $\alpha = 0.84$ and $p_e = 0.0008488$, when the distance-parameter is $a = 1$ or $a = 5$.

Solution:

$$a=1:\quad z_B = z_f \cdot \alpha \cdot C \approx 9807 \cdot 0.84 = 8238,$$

$$a=5:\quad z_B = z_f \cdot \frac{\alpha(1+0.84)(2+0.84)(3+0.84)(4+0.84)}{5!} \cdot C^{-5}$$

$$z_B = 9807 \cdot 0.6798 \cdot \frac{1}{1.0011} = 6659.$$

Comparatively 8247 or 6270 bursts were measured by sorting the registered sample.

In bursts with the parameter $a=5$, we found an average of 1.47 symbol errors.

2.2. Burst weight distribution

When we want to calculate how the number of errors is distributed within bursts, we start from Fig. 8.

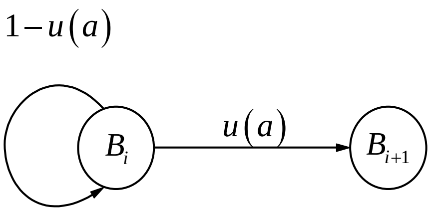


Figure 8: *Markoff-Diagram for calculating the Burst weight distribution*

Example 2:

Caculate the probability that a burst contains 4 symbol errors, when the distance parameter a has the value 5.

Use the model as given in Example 1

Solution:

According to Example 1, the value of 0.6791 was calculated for $u(5)$. With (39) we obtain

$$p\{g=4\} = 0.6791[1-0.6791]^{4-1} = 0.02244 \tag{40}$$

Only 2.244% of the bursts have 4 errors. These means 149 bursts from a total of 6659 (compare with Example 1).

.2.3 Burst length distribution

Under the assumption of independent gaps between symbol errors, the probabilities should be calculated for the burst length l (Fig. 9).

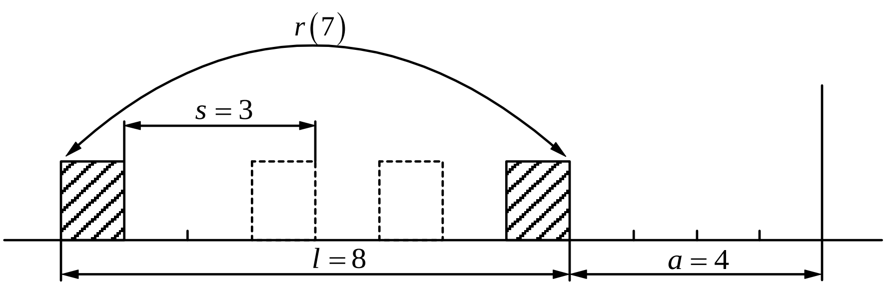


Figure 9: *Calculation of the burst length distribution; for* $r(7)$

The correlation between the first and last errors of the burst is demonstrated in Figure 8, where l is the burst length and a is the distance parameter.

First, the correlation is determined under the condition that the gaps between the error positions are always smaller than a.

Using the general approach for the generating function $R(t)$ in determining the correlation between the first and the last errors,

$$R(t)=\frac{p_e}{1-P(t)}=p_e\frac{(1-C\cdot t)^{\alpha}}{1-t} \tag{41}$$

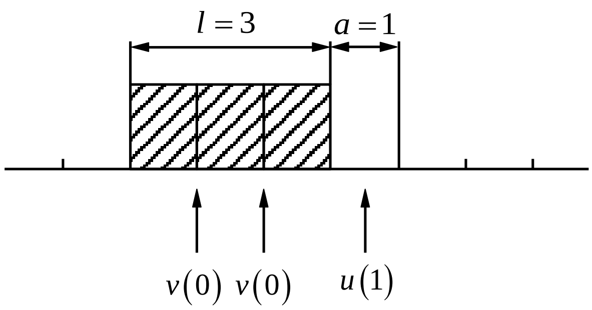


Figure 10: *The Burst of length 3 with 3 errors in Example 3*

Example 3:

Determine the probability that a burst has the length $l=3$ whereby $a=1$.

Solution

The required probability is the coefficient of $t^{l-1}=t^2$ in the expression (49)

$$P\{l=3,a=1\}=u(1)v_0^2$$

For this example, the solution is obtained intuitively (see Fig. 9).

Since the weight is also equal to 3, the same solution as in (39) must apply, considering that $1-u(1)=v(0)$

For the model (Example 1), it is given

$$u(1)=\alpha\cdot C=0,81113;\ v(0)=0,18868\ ;$$

$$P\{l=3,a=1\}=0,02888$$

From 7958 bursts (see Example 1), 230 have the weight 3.

A burst ends when **two error-free** symbols follow one another. It follows from (46),

$$B_2(t) = \frac{u(2)}{1 - v_0 t - v_1 t^2} \tag{50}$$

$$v_0 = v(0) = 1 - \alpha \cdot C,$$
$$v_1 = \alpha \cdot \left(1 + \frac{1+\alpha}{2} \cdot C\right) \cdot C$$

Example 4:

In A-model Example 1, the probability was calculated that a burst had the length $l = 5$,where no more then one error-free symbol occurs between two symbol errors in the burst!

Solution:

The burst is finished when two error-free steps occur consecutively

$$v_0 = 1 - 0{,}81145 \cdot C = 0{,}1885; \quad \text{where } C = 0.999836$$

$$v_1 = 0.81145 \cdot \left(1 - \frac{1 + 0.81145}{2} \cdot C\right) \cdot C = 0.076607$$

$$t_{1/2} = \frac{-0.1885 \pm 0.58477}{0.153214};$$

$$t_1 = +2.5864; \; t_2 = -5.0470;$$

$$u(2) = \frac{\alpha(1+\alpha)}{2} \cdot C^2 = 0.7347;$$

$$P\{l, a = 2\} = 1.256\left[2.5864^{-l} - (-5.0470)^{-l}\right].$$

Intuitively, $P\{l = 1, a = 2\} = u(2)$

For $l = 5$, we obtain

$$P\{l = 5, a = 2\} = 1.256\left[2.5864^{-5} - (-5.0470)^{-5}\right],$$
$$P\{l = 5, a = 2\} = 0.01124.$$

In a sample with 9807 errors (Example 1), there are 7577 bursts, when $a = 2$ is selected, whereby only 85 bursts have the length $l = 5$.

Figure 10 shows the probabilities for the burst length l.

It is shown that the alternating portion of t_2 in (57) disappears quickly, so that the function for longer lengths is linear when given semi-logarithmically.

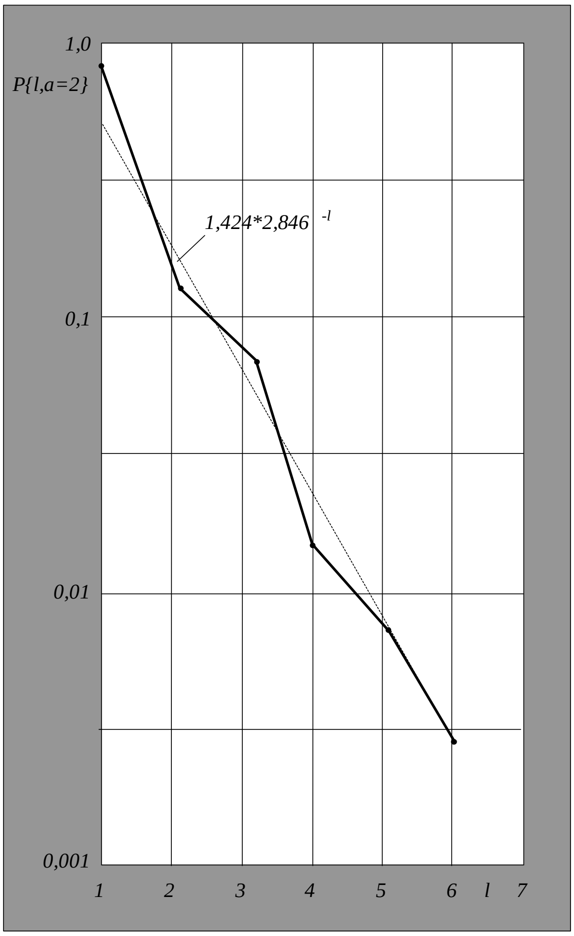


***Figure 11**: Probability for the burst length l in bursts where $a = 2$*

A-Model: $\alpha = 0.84$; $p_e = 0.0008488$

2.4 Distribution of Bursts with length l and weight g

When using block-free code, it is necessary to estimate the reduction factor for symbol errors. The reduction factor is understood to be the ratio of undetected symbol errors to the total number of symbol errors, regardless of coding. From the properties of the code used, we can calculate how many bursts of length l containing g symbol errors cannot be detected.

Example 5:

Using the Burst Analysis Matrix in Tab. 4 of an artificially generated error sequence (a = 5)

a) Calculate the number for the occurrence of all burst-weights.

The number of bursts with single errors $(g=1)$ in this sample is 3446. Then follows

$u(5)=3446/5727=0.5843$, and we get $\mathrm{Num}\{g\}=u(5)[1-u(5)]^{g-1}\cdot 5727$ for

$g=$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**>8**	
	3446	1372	546	217	87	35	14	5	5	calculated
	3446	1375	528	219	108	34	12	2	3	randomly chosen, Tab.4

b) Calculate the distribution of length l in all bursts with weight $g=2$

It is clear that the longest burst arises, if between the three symbol errors $a-1=2$ error-free steps follow each other. We then obtain $l_{\max}=7$.

By arranging (64) in powers of t, we obtain, using values $v(\mathrm{x})$ and α as calculated (20),(25),(29) with EXCEL

$$\left[F^*(t)\right]^1=\frac{1}{\left[1-u(5)\right]^1}\left[v(0)t+v(1)t^2+v(2)t^3+v(3)t^4+v(4)^5\right]^1.$$

$$\mathrm{Num}\{l=2/g=2;a=5\}=1372\cdot\frac{v(0)}{(1-u(5))}=712$$

$l=$	**2**	**3**	**4**	**5**	**6**	
	712	282	196	118	90	calculated
	796	238	156	96	89	randomly chosen, Tab.4

c) Calculate the distribution of length l in all bursts with weight $g=3$

$$\left[F^*(t)\right]^2=\frac{1}{\left[1-u(5)\right]^2}.[v(0)t^1+v(1)t^2+......+v(4)t^5]^2$$

$$Num\left\{l=5/g=3;a=5\right\}=528\cdot\frac{2v(0)v(2)+v(1)^2}{\left[1-u(5)\right]^2}=528\cdot\frac{0.02935}{0.1729}=89.6$$; randomly 85

The desired result $p(l/g=3;a=5)$ is the coefficient of t^{l-1} from the power series. It is evident that the formal solution is easily programmable for any values of g and a .

2.5 Burst Analysis Matrix

The previously calculated probabilities can be represented clearly in a two-dimensional probability table for bursts, with features *length* l and *weight* g and a given distance parameter a. This table lso contains the number of measured and sorted events and is called the **Burst Analysis Matrix.** The matrix shown in Table 4 was created by sorting an artificially generated error sequence.

length l	weight g									
	1	2	3	4	5	6	7	8	9 - 12	$\sum l$
1	3446									3446
2		796								796
3		238	146							384
4		156	132	38						326
5		96	85	40	7					228
6		89	69	41	11	0				210
7		0	56	30	14	3	0			103
8		0	24	34	19	1	2	0		80
9 - 12		0	16	34	47	25	5	1	0	128
13 - 18		0	0	2	10	4	5	1	3	25
19 - 25		0	0	0	0	1	0	0	0	1
$\sum g$	3446	1375	528	219	108	34	12	2	3	5727

Table 4: *Burst Analysis Matrix of an artificially generated error sequence (a = 5)*

The column totals are the number of bursts with weight g, and the line totals are the number of bursts of length l. For example, 40 bursts were registered with errors of weight $g = 4$ and length $l = 5$. As the burst-weight distribution and burst-length distribution may be calculated with (40), (46) respectively, the approach (63) is used to split the column sum of bursts where the weight g is constant, according to burst length l. When all elements in Table 4 have been calculated in this way, the results can be double-checked using the sums of the rows. The given dividing line indicates the maximum possible burst lengths.

3. Calculation of error structures in blocks

In many error control methods, the data are encoded and transmitted in blocks. A block is a group of symbols which is encoded and transmitted as a unified whole, in order to determine transmission errors. Unrecognisable error structures should occur sufficiently rarely. Therefore, it is of interest to calculate with certainty the probabilities of non-detectable error structures despite the relevant coding.

Some introductory remarks illustrate the need for, and the potential applications of, such calculations.

In the development of various codes, the binary channel has been assumed to be "memory-less".

The resultant classical works of coding theory, for example Peterson [9], have gained wide acceptance.

Example:n=5bits, pe=0.000258430	*measured UHF 600 bit/s*	*calculated according to Bernoulli*
weight g	*number measured*	*number calculated*
1	*2373*	*3191*
2	*294*	*1.649*
3	*54*	$4.26 \cdot 10^{-4}$
4	*11*	$5.5 \cdot 10^{-8}$
5	*4*	$2.8 \cdot 10^{-12}$

Table 5: *Measured and calculated weight distributions in disturbed blocks*

However, in consideration of Table 5, it can be seen that in "memory-less" channels within disturbed blocks, almost always single errors occur, provided that the symbol error probability is << 0.5.

Therefore, codes have been developed to correct *single errors*. Later, actual measurements revealed that the proportion of multiple errors is in fact significantly higher in disturbed blocks, as suspected. We define **bundle errors** between the first and last error with length l and g errors in blocks of length n.

Cyclical codes detect bundle errors in blocks up to a certain length l.For longer bundle errors, the probability of undetected error blocks can be estimated. These codes identify further error blocks up to a weight g.

Channel measurements described in the literature therefore focus on specifying the length distribution and the weight distribution of bundle errors within disturbed blocks.

It has been noted that there are particular short bundle errors with certain weights which remain particularly frequently undetected by the code.

Therefore, it is proposed as in [8], to sort the bundle error occurring in blocks according to the two features length l and weight g, and to tabulate them by way of a so-called channel matrix (Table 6), as in Table 4.

l length	weight g										$\sum l$
	1	2	3	4	5	6	7	8	9 - 12	13 - 15	
1	3469										3469
2		750									750
3		214	141								355
4		133	128	32							293
5		77	76	36	6						195
6		67	59	30	11	1					168
7		55	42	21	8	2	0				128
8		37	33	25	12	3	1	0			111
9 – 12		74	88	54	33	14	2	2	0		267
13 - 15		19	19	17	8	5	2	0	0	0	70
$\sum g$	3469	1426	586	215	78	25	5	2	0	0	5806

Table 6: *Channel matrix of an artificially generated error sequence for bundle errors of length* l *and weight* g *in blocks of length* $n = 15$

Not all coding theorists or developers of transmission equipment have had access to results of error structure measurements.

Therefore in the following section, probability formulas are derived for the occurrence of elements in this channel matrix and in the illustrative examples, based on the A-model.

3.1. Single errors in blocks of length n

A single symbol error can be located in each of the positions of the block (Fig. 12).

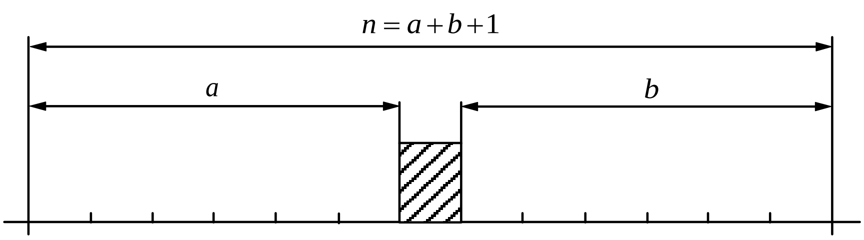


Figure 12: *Calculation of the probability of single errors in a block of length n*

Example 6:

What is the probability that a single error occurs when we have a block length $n=1023$ or the block length $n=5$?

Use the A-model again as per the measured sample in Example 4 whereby

$p_e = 0.0008488$, $\alpha = 0.84$, so is $C = 0.999779$, the number of transmitted symbols is 11,554,541 and the number of symbol errors $z_f = 9807$.

Solution for $n=1023$

From (69): $p\{n, g=1\} = \frac{p_e}{\Gamma(2\alpha)}(n-1)^{2\alpha-1} \cdot C^{n-1}$ we get

With Excel: $\Gamma(x) = Exp(GammaLn(1.68))$

$$p\{1023, g=1\} = \frac{p_e}{\Gamma(1.68)}(1023-1)^{0.68} \cdot C^{1022} = \frac{0.0008488}{0.905} \cdot 111.2824 \cdot 0{,}79781 = 0.083327$$

From $11.554.541/1023 = 11.295$ transferred blocks, each of 941 of these blocks should contain a single error. However, in all, 1096 blocks containing a single error were actually detected in our sample.

Solution for $n=5$ we obtain from (68)

$$p\{n, g=1\} = p_e \frac{2\alpha(2\alpha+1)\cdots(2\alpha+n-2)}{(n-1)!} C^{n-1}$$

$$p\{5, g=1\} = p_e \frac{1.68 \cdot 2.68 \cdot 3.68 \cdot 4.68}{4!} \cdot 0.99912 = 0.00274$$

From 11,554,541/5 = 2,310,908 transferred blocks, 6,332 blocks should therefore theoretically each contain a single error. In practice, 6,271 blocks were found to each contain a single error in our sample.

This demonstrates how the A model provides good estimates.

Using a single error correction code, 10% of symbol errors will be sure to be corrected for a block length of $n = 1023$ bits. However, 64% of symbol errors will be corrected for a block length of n = 5 bits.

3.2. Distribution of error length in erroneous blocks

Different codes have the capacity to detect errors in disturbed blocks provided the length l of the error bundle in the block does not exceed a certain value.

Such a bundle error is shown in Figure 13.

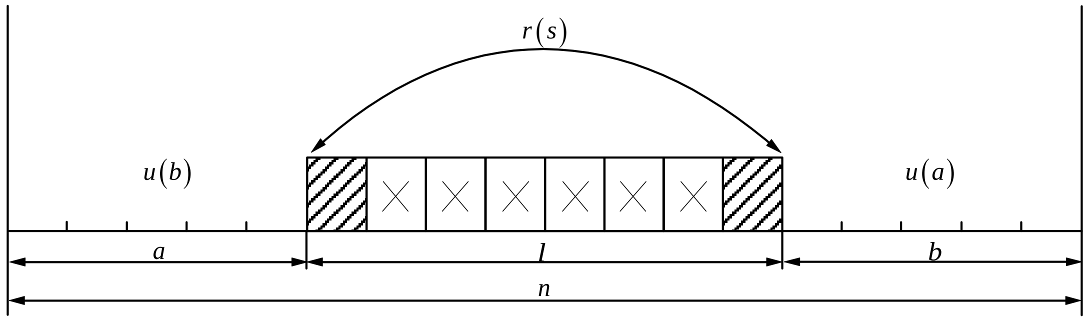


Figure 13: *Error bundle of length l in a block of length n*

Example 7:
Calculate the probability for the proportion of error bundles with a length of $l > 3$ in disturbed blocks of length $n = 5$ for the A-model according to Example 1.

Solution:
Where $\alpha = 0.84$ und $p_e = 0.0008488$ and $\beta = \dfrac{1}{4486}$ $\quad C = (1 - p_e^{1/\alpha}) = 0.99978$

$p_b(5) \approx p_e n^{\alpha} = 0.00328$ or more precisely:

$$p_b(5) = p_e \sum_{k=0}^{4} u(k).$$

Recursively it follows from (20), Chapter 1

$$u(0) = 1;. u(1) = \alpha \cdot C; \; u(2) = u(1)\frac{1+\alpha}{2}\mathrm{C}^2;$$

$$u(k) = u(k-1)\frac{k-1+\alpha}{k}\cdot \mathrm{C}, \quad p_b(5) = 0.003435$$

However a block error frequency of 0.0033926 was measured in finite sample.

According to Example 6, the probability of a single error is $p\{5, g = 1\} = 0.00274485$.
From (73), it follows:

$$p\{l=2,n=5\} = p_{\mathrm{e}} \frac{1.68\cdot 2.68\cdot 3.68}{3!}\mathrm{C}^3\cdot(1-0.84\cdot C),$$

$$p\{l=2,n=5\} = 0.000375,$$

$$p\{l=3,n=5\} = p_{\mathrm{e}} \frac{1.68\cdot 2.68}{2!}\mathrm{C}^2\cdot\left(1-0.84\cdot C-\frac{0.84\cdot 0.16}{2}\mathrm{C}^2\right),$$

$$p\{l=3,n=5\} = 0.0001773.$$

Thus, the proportion of blocks with error bursts $l>3$ in disturbed blocks of length $n=5$ is

$$\frac{P\{l>3,n=5\}}{p_b(5)} = 1-\frac{0.00274485+0.000375+0.0001773}{0.003435} = 0.040131.$$

Only 4% of erroneous blocks where $n=5$ have error bundles of length $l>3$.

This corresponds to just 319 of all $2,310,908\times 0.003435 = 7938$ erroneous blocks. In the measured sample of 7840 blocks, 426 blocks were found to have error bursts $l>3$.

It is seen that in the A-model where there are independent gaps between symbol errors, longer error bundles occur less frequently than in the measured sample. Real samples reveal that after one short gap another short gap is more likely to follow than a large gap. This memory feature need only be considered in more complicated models [2].

Table 7 shows the comparison of measured and calculated values.

length l of error bundles in block of length 5	*measured*	*Calculated with the A-Modell*
1	*6271*	*6345*
2	*680*	*861*
3	*463*	*408*
4	*266*	*220*
5	*160*	*104*
Summe	*7840*	*7938*

Table 7: *Length l of error bundles in block of length n=5*

Example 8:

Using the A-model with the sample in Example 1, calculate the probability that in a block of length $n=5$, the first and the last symbols are incorrect and a total of 3 errors occurs !

Solution:

The first error occurs in position 1 with the probability p_e ("history" unknown).

The second error can be located at any of positions 2, 3 or 4, whereas the last error occurs in the 5th position.

Therefore, three different single patterns must be calculated
Thus follows:

$$P\{M\} = p_\mathrm{e}\left(v_0 v_2 + v_1 v_1 + v_2 v_0\right) = p_\mathrm{e}\left(v_1^2 + 2 v_0 v_2\right) = p_e \cdot 0.1778707 \tag{91}$$

For larger error bundles, using this formula would require us to calculate far too many summands. Therefore, the solution is best given by using formula (90): $P\{M\} = p_\mathrm{e} \cdot p\left(l=5, g=3\right)$.

Calculation scheme for (90):

$$\begin{array}{lll}
r & j & \\
1 & 3: & +\binom{2}{1}\binom{1}{1}u(1,3) \\
 & 4: & -\binom{2}{1}\binom{1}{0}u(1,4) \\
2 & 2: & +\binom{2}{2}\binom{2}{2}u(2,2) \\
 & 3: & -\binom{2}{2}\binom{2}{1}u(2,3) \\
 & 4: & +\binom{2}{2}\binom{2}{0}u(2,4)
\end{array}$$

$$P\{M\} = p_\mathrm{e}\left[2u(1,3) - 2u(1,4) + u(2,2) - 2u(2,3) + u(2,4)\right]. \tag{92}$$

In contrast to the elementary calculation from (91), only 5 distribution functions $u(r, j)$ need to be determined. Since need to be inserted. However, the approach (90) for long error bundles proves indispensable.

$$+2u(1,3) = \frac{2}{3!}\alpha(\alpha+1)(\alpha+2)C^3 = 2u(3) \qquad = +1.46219942$$

$$-2u(1,4) = -\frac{2}{4!}\alpha(\alpha+1)(\alpha+2)(\alpha+3)C^4 = -2u(4) = -1.403401634$$

$$+u(2,2) = \frac{2}{2!}\alpha(2\alpha+1)C^2 \qquad = +2.250206399,$$

$$-2u(2,3) = -\frac{2}{3!}2\alpha(2\alpha+1)(2\alpha+2)C^3 \qquad = -5.519287951$$

$$+u(2,4) = \frac{2}{4!}\alpha(2\alpha+1)(2\alpha+2)(2\alpha+3)C^4 \qquad = +3.228070837$$

$$\overline{\qquad 0.01778707}$$

where $\alpha = 0.84$ $p_e = 0.0008488$ and $C = (1 - p_e^{1/\alpha}) = 0.999779239$ according to example 1.

When an error occurs in position 1 as in well as position 5, the probability that two further errors occur within the three interim positions is 1.78%.

Considering that the likelihood that the first symbol error occurs at position 1 is only $p_e = 0.0008488$, we can see the "memory" of the channel as reflected in the A-model.
In the sample from Example 1 with 11,554,541 transmitted bits,

$$z_M = \frac{11554541}{5} p_e \cdot p(l=5, g=3) = 35$$

error patterns of this kind must occur.
45 such patterns were measured.

If, however, the normal distribution for the measured events is to be believed (based on large trial numbers), then event numbers will only occur outside the 3σ boundaries with a so-called error probability of $\varepsilon = 0.0027$. See Part 1, Koller [10].

The Normal distribution is assumed; a deviation of

$$\Delta z \approx \pm 3\sqrt{p(1-p)n}$$

in the measurement is allowed.

With $1-p \approx 1$ and $np = 35$, we get $\Delta z \approx \pm 17$. The resulting calculation will not be rejected with an error probability of 0.27% for the real channel, as long as 35 ± 17 events have been measured.

3.4. Distribution of weight g of error bundles of length l into erroneous blocks

If we divide the error bundles occurring in erroneous blocks into classes, we can determine the proportion of error bundles not recognizable by an error-protection code within each class.

We can assign the appropriate weight to each probability of occurrence per class, and determine the average over all classes.

The result is the probability of the occurrence of blocks that contain undetected error bundles despite having an error-protection code. This important result can be divided between calculations for the channel side and calculations for the code side. The probability of an error bundle class occurring depends on the channel characteristics and are calculated as follows.

The calculation depends on the proportion of undetectable error bundles of the code used. See [2], [8].

Fig. 15 represents how we calculate the probability $p\{l,g,n\}$, that in a block of length n an error bundle occurs of length l with g symbol errors.

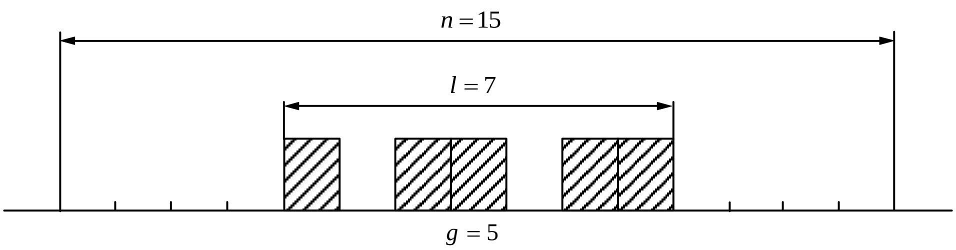


Figure 15: *Calculating the probability of the occurrence of error bundles of the length l with g symbol errors in block of length n*

Example 9:

Calculate the probability $P\{l,g,n\}$, for error bundles of length $l=5$ with $g=3$ symbol errors occurring in blocks of length $n=63$. Here, the A-model is used according to Example 1.

Solution:

For the factor $u(2,n-l)$, the approximation (69) applies for large values of $n-l$.

$$u(2,n-l)\approx\frac{1}{\Gamma(2\alpha)}(n-l)^{2\alpha-1}\cdot \mathrm{C}^{(n-l)},\quad \text{whereby}\quad C=(1-p_e^{1/\alpha})=0.999779 \tag{96}$$

$$u(2,58) \approx \frac{1}{\Gamma(1.68)} \cdot 58^{0.68} \cdot C^{58},$$

$$u(2,58) \approx 1.11279015 \cdot 15.8172521 \cdot \frac{1}{1.01290} = 17.3770$$

with the help of EXCEL: $\Gamma(\alpha) = Exp(GammaLn(\alpha))$

The second factor was calculated in **Example 8** as $p(l=5, g=3) = 0.017788.$

With $p_e = 0.0008488$ (from Example 8), we obtain the desired probability:

$$P\{l=5, g=3, n=63\} = p_e \cdot 17.2535 \cdot 0.017788 = p_e \cdot 0.30690.$$

Thus, $z_M = 48$ such patterns.

$$z_M = \frac{11,554,541}{63} \cdot p_e \cdot 0.3069 = 48; \Delta z_M \approx \pm 3 \cdot \sqrt{48} = \pm 21$$

must be included in the sample with 11,554,541 transmitted symbols.

However, only 18 patterns of this type were registered.

This deviation is not random, but rather it refers to the tendency, that in the A-model, assuming independent gaps, short errors bundles occur more frequently while long error bundles are rarer than is the case with real channels.

Burst process models (HMM) are by definition more accurate.

The construction of such burst process models (HMM) is described by Turin [15] and Wilhelm [2]

3.5. Distribution of weight g of error bundles in erroneous blocks (weight spectrum)

In coding theory, the channel was described initially by specifying the distribution of the number of symbol errors in erroneous blocks.

For n different classes, the probabilities $p\{g,n\}$ can be calculated using formula (95).

Example 9:

Calculate the probability, $\mathrm{p}\{g,n\}$ for all error bundles with $g=2$ symbol errors occurring in blocks of length. $n=3$ using formula (98).

Solution:

$$p\{g=2, n=3\} / p_e = u(2,1) \cdot [\mathrm{u}(1,0) - \mathrm{u}(1,1)] + u(2,0) \cdot [u(1,1) - u(1,2)]$$

$$= 2 \cdot \alpha \cdot C \cdot v(0) + v(1) = 2 \cdot u(1) \cdot v(0) + v(1)$$

That's plausible because there are three different pattern exists:

$LL0 \triangleq v(0) \cdot u(1)$; $L0L \triangleq v(1)$; $0LL \triangleq v(0) \cdot u(1)$

A. Appendix – Basic set of formulas for the A-model

A.1 Definitions

(1)	Number of error-free symbols between two symbol errors	k
(2)	Error distance	$s = k + 1$
(3)	Symbol error probability	p_e
(4)	Bundling factor	$1 - \alpha$
(5)	Limiting factor for including the BSC-Channel	$C = (1 - p_e^{1/\alpha})$
(6)	Block length	n
(7)	Number of symbol errors in the block, burst or bundle	g
(8)	Error burst length in bursts or error bundle length in blocks	l
(9)	Number of error free symbols to separate two bursts	a

A.2 General relationships for the A-model

A.3 Burst relationships

A.4 Disturbed block relationships

Bibliography

[1] *Wilhelm, C.:* Der symmetrische Binärkanal mit Gedächtnis – eine Systematik der Kanalmodelle. Nachrichtentechnik 19 (1969) II.10, S. 374-378, 383 und 384.

[2] *Wilhelm, C.:* Datenübertragung. Berlin: Militärverlag 1976, 684 S.

[3] *Johlig, E.:*, u. a.: Fehlerstrukturmessungen bei der Datenübertragung mit höheren Geschwindigkeiten. Nachrichtentechnik 19 (1969) II. 3, S. 115-117.

[4] *Hoffmann, G.:*, u. a.: Messtechnische Probleme bei Fehlerstrukturuntersuchungen im PCM-Kanal. Vortrag zum 6. Fachkolloquium Informationstechnik in der technischen Universität Dresden, Sektion Informationstechnik, vom 17. bis 19.4.1973.

[5] *Janke, E.:*, *Emde, F.*, und *Lösch, F.:* Tafeln höherer Funktionen. Moskau: Verlag Nauka 1968.

[6] *Lochmann, D.:* Statistische Untersuchung der Kombination von Stördetektor und Codierung bei der Datenübertragung im Fernsprechnetz am Beispiel von 200 bit/s. Dissertation, TU Dresden 1971.

[7] *Gilbert, E. N.:* Capacity of a burst-noise channel. Bell Syst. Techn. J. 39 (1960/II) II.9, S. 1253-1265.

[8] *Wilhelm, C.:* Über den Zusammenhang zwischen Kanal und Codierung bei der Datenfernübertragung. Nachrichtentechnik 17 (1967), H. 10, S. 386, 391 und 392.

[9] *Peterson, W.:* Prüfbare und korrigierbare Codes, München 1967.

[10] *Koller, S.:* Graphische Tafeln zur Beurteilung statistischer Zahlen. Verlag Theodor Steinkopf, Dresden und Leipzig 1943.

[11] *Wilhelm, C.:* Konstruktion endlicher Kanalmodelle des symmetrischen Binärkanals mit Gedächtnis bei vorgegebenen nachrichtentechnischen Forderungen
Dresden, Technische Universität, Fak. Datenverarbeitung; Dissertation A, 1973

[12] *Wilhelm, C.*: Beiträge zur Bemessung von Nachrichtensystemen und zur Berechnung der Störfestigkeit
Dresden, Hochschule für Verkehrswesen, Dissertation B, 1989

[13] *Ahrens, A.; Wilhelm, C.:* Bestimmung des Reduktionsfaktors bei Einsatz zyklischer Codes auf Kanälen mit gebündelt auftretenden Übertragungsfehlern; Rostock 2001; 10. Symposium Maritime Elektronik, Arbeitskreis Maritime Mess- und Informationselektronik, pp. 141-144,

[14] Wilhelm-Modell: Lehrstuhl Nachrichtentechnik, Technische Universität München:
www.lntwww.de/downloads/6_Digitalsignaluebertragung/Theorie/Dig_Kap5_gesamt.pdf
pp 19-27 (aktuell 2014)

[15] Turin, William: Performance Analysis and Modelling of Digital Transmission Systems,
Kluwer Academic/Plenum Publishers 2014

[16] Proakis, John G.: Digital Communications, Fourth Edition, McGraw-Hill Higher Education 2001