A brief history of the channel models of binary symmetric channels with memory

From the *Gilbert model* to the *A-model* for the calculation of error structures with variable parameters, so they do not have to be sorted out ***during or from*** measurement.

Suitable for the evaluation of procedures and codes without simulation or field trial!

During the transmission of data undetected errors occur despite coding.Transmission procedures suddenly leave the regular course, they can even block themselves.
The currently standardized methods (Internet, mobile telephony) have been nearly optimized empirically and basically satisfy the requirements concerning data transmission.
The signal distortions and falsifications are currently modeled in many scientific papers and examined with the result that on the one hand, the modulation processes could be optimized, but on the other hand, the order of bit errors, which occur stochastically in the digital output remains unclear.

Fifty years ago the "father" of the channel models, E.N. Gilbert, published “Capacity of a Burst-Noise Channel”, BSTJ 39 (1960/II), No. 9 S 1253-1265 showing a simple model of the binary symmetric channels with memory

Gilbert’s model contained the following assumptions

- The memory only extends back to the previous bit error
- The consecutive error-free gaps are independent
- The distribution ***u (k)*** of the gap length ***k*** is given,
  i.e. the probability that the next gap is **> *k:***

$$u(k) = AJ^k + (1 - A)L^k$$

with the parameters ***A, J*** and ***L***

Afterwards, many authors noted that this distribution contradicts the measured results, (Gilbert had considered memory length only up to 40 bits), whereby these later authors approximated distributions with different formulas (Merz, Berger, ZNIIS Moscow, Abramenko, Dombrowski, Muller, Swoboda, and others).

Later still it was realized that the channel memory extends beyond the last bit error. This led to the class of models called hidden Markov model (**HMM**).
The classicist William Turin published in his 1999 book, “Digital transmission performance analysis and modeling of systems”, ISBN 0-306-48191-X ,Kluwer Academy / Plenum Publishers, NY 2004, his theory concerning such HMM models.

These opposing models presented a dilemma. It was only possible to calculate with great effort or not at all the probabilities for sums of events from these distribution functions, sums for which the evaluation of codes and procedures is required,
for example:

- Number of ***g*** errors in error bursts of length ***L***
  where a burst is completed, provided at least ***a*** error-free steps follow;
- Number ***g*** of errors in blocks of length ***n***,

• The number of bursts of length ***L*** with ***g*** errors in blocks of length ***n***
• number of erroneous blocks of length ***n***,
• Burst weight distribution, burst length distribution
• etc.

Why is this issue still relevant today?
Surprisingly, in recent works historical models are mostly used, because measured results of error- structures were not available. Therefore the above issue is still current. However, there are hidden Markov models constructed from measurements, all without manageable formulas for probabilities for sums of the above events. However, measured error sequences can be sorted and thus the frequencies of events can be listed, with a confidence interval for each respective finite sample measured.

How can properties of codes and procedures be compared?
The length of any two tree trunks can be easily compared with a simple stick without *calibrated scale*. With a channel model with manageable formulas for the above sums of events is a comparison of codes or procedures possible when the model is based on real measurements.

The A-model (distance model), the L-model (gap model)
From measurements of error structures carried out over many years (consecutive error-bits) by the Central Office of Communication Networks (ZFN, Königs Wusterhausen, Germany) between 1965 and 1986, on channels of 200 bit /s to 2048 Mbit / s together with their statistical evaluations have been empirically substantiated by several scientific studies. The following result provides such confirmation, as do results from other authors:

The block error probability p_b as a funktion of the block length n is **linear** in double-logarithmic representation in the initial part of

$$n \leq n_o$$

$$\log[\, p_b(n) \,] = \log p_e + \alpha \log n$$

with the bit-error probability p_e and the bundling factor α
It then follows:

$$p_b(n) = p_e n^{\alpha}$$

for

$$0,5 < \alpha < 0,95$$

This is the first ever formula which has led to a complete set of *Generating Functions* for the above error structures as *sums of events* with two parameters, probability of bit-error and bundling factor.
By time-varying these parameters, one can simulate the effects of interference, fading and noise on the error structures. Field tests are only necessary in conclusion.
These formulae were derived, implemented and published in the book "*Datenübertragung*" edited by *Wilhelm, Claus, Militärverlag Berlin 1976.*
This website gives step-by-step examples of these models using the above formulae.